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AUTHOR	Armstrong, Jenny R.
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ABSTRACT

Mathematical learning at six cognitive levels, in areas of exact and varied repetition, was studied in 28 educable mentally handicapped students (mean IQ 74, mean age 12.6). Four different machine-presented programs of 10 lessons each utilized inductive or deductive modes of presentation and exact or varied forms of curriculum repetition. Results indicated that number learning at the knowledge, application, and evaluation levels was better facilitated by an exact form of curriculum repetition. Mathematical learning of operations was equally facilitated by inductive and deductive modes of presentation. Both area and topical curriculum organization were equally effective, regardless of the mode of presentation used; however, exact repetition better facilitated learning at the comprehension level when implemented by the inductive mode, while varied repetition was more effective when implemented deductively. (Author/JD)

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Jenny R. Armstrong
University of Wisconsin
Madison, Wisconsin

April, 1968

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U.S. DEPARTMENT OF
HEALTH, EDUCATION, AND WELFARE

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INTRODUCTION

The purpose of the second phase of the study was to further examine the instructional modes of presentation, inductive and deductive, when implemented by teaching machines rather than teachers. During the first phase of the study, less difference was found between the inductive and deductive groups than was expected. It was suggested that this could have been due to the lack of control over teachers in their "pure" use of the modes of presentation. Thus, the second phase of the study was designed so that the modes of presentation could be implemented by means of teaching machines rather than teachers in order to increase control over the two forms of curriculum implementation.

Since the area spiral form of curriculum organization was consistantly found to better facilitate mathematical learning than the topical spiral form, the area spiral form was exclusively used in this phase of the study (Armstrong, 1968[a]; pp. 84 and 91).

The sample for this phase of the study was made up of 32 Educable Mentally Retarded (EMR) pupils. Consequently, both the level of the content and the amount of content covered in the program was reduced from that covered in phase I. There were ten lessons written on the following concepts: set membership and definition, cardinality of sets, subset relations, set equivalence, set union, phase value, and number relations. The geometry area and the topic of properties were both deleted.

Although the form of curriculum organization was held constant, the curriculum form of repetition was varied. The repetition of presentation component has been examined in the development and use of film curriculums for the EMR. One film producer has developed an arithmetic film curriculum on counting to three, <u>One and Two and Three</u> (Wexler, n.d.) which completely repeats itself. This approach to repetition in curriculum has been examined experimentally by Brannan (1965). Brannan (1965) compared five different repetition conditions using films. A sample of 285 EMR's with IQ range of 50-78 and CA from 11-14 were randomly assigned to five different repetition conditions, pupil participation, teacher presentation, discussion and film reshowing, and film reshowing. When repetition and non-repetition groups were compared, the learning of the repetition groups was found to be significantly superior to the non-repetition groups both with respect to immediate recall and retention. No differences were found, however, among the five different types of repetition: discussion, pupil participation, teacher presentation, discussion coupled with film reshowing, and film reshowing.

Two forms of repetition were examined in this study, exact and varied. The exact form of curriculum repetition involved the exact repetition of the first half of a 15 minute lesson whereas the varied form of curriculum repetition used the first half of the 15 minute lesson and during the second half of the 15 minute lesson utilized new and different examples to teach the same content. As in phase I, the mathematical learning of the subjects was assessed globally as well as

at various cognitive levels, in different mathematical areas, and on various mathematical topics. The cognitive levels examined were knowledge, comprehension, application, analysis, synthesis and evaluation. The areas were sets and numbers; the topics were terminology, relations and operations. As stated earlier, geometry and properties were deleted from the curriculum of this phase of the study. Therefore, the two variables manipulated in this phase of the study were the instructional mode of presentation (inductive or deductive) and the curriculum form of repetition (exact or varied). The materials were presented on teaching machines.

The Ken Cook Company Mark VII teaching machine was used. It has options for syncronized visual-audio stimulus presentation. It also permits multiple choice responses and visual and audio reinforcement based on the student's answer choice. In this way, by using pretaped lessons it was hoped that many of the inherent problems which arose when the teacher variable was introduced could be avoided. It was also felt that by using colorful high interest stylized drawings the pupil's attention span could be increased.

Thus, the purpose of this phase of the study was to determine the relative effect of two forms of curriculum repeptition, exact and varied, and two instructional modes of presentation, inductive and deductive, on mathematical learning globally, at each of six cognitive levels, in two areas, and on three topics.

The hypotheses of this phase of the study stated in null form are as follows:

1. There is no difference in the total mathematical learning of EMR's in either an exact or varied form of curriculum repetition.

2. There is no difference in the total mathematical learning of EMR's taught by either an inductive or deductive mode of presentation.

3. There is no difference in the total mathematical learning of EMR's in either an exact or a varied form of curriculum repetition presented either inductively or deductively.

4. There is no difference in the mathematical learning at various cognitive levels of EMR's in either an exact or a varied form of curriculum repetition.

5. There is no difference in the mathematical learning at various cognitive levels of EMR's taught by either an inductive or a deductive mode of presentation.

6. There is no difference in the mathematical learning at various cognitive levels of EMR's in either an exact or varied form of curriculum repetition presented either inductively or deductively.

7. There is no difference in the mathematical learning in two

areas of EMR's in either an exact or a varied form of curriculum repetition.

8. There is no difference in the mathematical learning in two areas of EMR's taught by either an inductive or deductive mode of presentation.

9. There is no difference in the mathematical learning in two areas of EMR's in either an exact or varied form of curriculum repetition presented either inductively or deductively.

10. There is no difference in the mathematical learning on three topics of EMR's in either an exact or varied form of curriculum repetition.

11. There is no difference in the mathematical learning of topics by EMR's taught by either an inductive or deductive mode of presentation.

12. There is no di˜ference in the mathematical learning of topics by EMR's in either an exact or varied form of curriculum repetition presented either inductively or deductively.

II. PROGRAMS AND PROCEDURES

Experimental Program

The experimental program of this phase of the study consisted of ten lessons, each fifteen minutes in length with from 40 to 60 slide changes in each. The 35 mm color slides were produced from original artwork on transparencies. All of the slides for the set lessons were photographed on a gold background and all of those for the number lessons were on a green background. In general, the objects illustrated in the slides were drawings of every-day objects which had been suggested for teacher use as illustrative examples in the lessons of phase one of the study. To provide some identification source for the children, however, a character, Mr. Mathematics, was also added.

The form the experimental program took is best illustrated by an actual script. The script for lesson one is shown in Appendix I. Both an inductive and deductive script were written on the specified content of each lesson. The numbers in the script are the sequential number of the slide in the lesson and the identification number of the slide. A description of the slides used in lesson one is shown in Appendix II.

The four different treatment conditions, inductive-exact (I-E), inductive-varied (I-V), deductive-exact (D-E), and deductive-varied (D-V), were all derived from the two scripts written for each lesson. The (I-V) and (D-V) treatment conditions utilized the inductive and deductive scripts in their original form. (see Appendix I). The (I-E) and (D-E) treatment conditions, however, made use of a modified form of the original lessons. The first half of the script, inductive and deductive respectively, was completely repeated in presentation and the second half omitted. The dividing points in the two scripts, inductive and deductive, for lesson one are shown with four red stars marked in the script (see Appendix I). Each lesson consisted of two parts. In the exact repetition condition, the subjects viewed a seven and one half minute lesson, presented either inductively or deductively, two times. In the varied repetition condition, during the second seven and one half minutes, the subjects viewed different examples designed to reinforce the mathematical concepts introduced in the first half.

The 40 scripts, 10 lessons of four different treatment conditions, were all recorded by a single person experienced in educational radio and television announcing and broadcasting. The original tapes were then edited and programmed with inaudible "bleeps" to trigger slide changes at the appropriate times in the program sequence. The tapes were wound onto Cousino tape cartridges for use on the Ken Cook Co. Mark VII Teaching Machine.

Four Ken Cook Company Mark VII teaching Machines were used. The machines are equipped for syncronized visual-audio stimulus presentations and immediate questioning with feedback capabilities. The machines employ an Argus slide projector and Cousino tape deck. There is a multiple choice button arrangement which permits the child to respond. This

button system utilizes three colors. Each machine has its own ear phones and an extra phone jack to permit listening by proctors. Each machine was set up on a folding table. These tables were separated by folding screen dividers.

Experimental Procedures

In order to determine the relative effects of the four treatment conditions, (I-E), (I-V), (D-E), and (D-V), on mathematical learning of EMR's, a sample was drawn, certain content tests were developed, and information on the variability between subjects within groups on prior mathematical knowledge and general mental ability was obtained.. The basic experimental design of the study followed closely that of phase I (see Armstrong, 1968[a]).

The Sample

Thirty-two educable mentally retarded pupils with a mean age of 12.6 and a mean IQ of 74 served as subjects. Of these 19 were males and 13 were females. They were drawn from four classes in the Madison Public School System on the basis of teacher recommendations. The teachers were asked to eliminate those children who had severe emotional disturbance, or physical impairments which would prohibit their full utilization of the teaching machines.

Experimental Design

A 2x2 completely crossed and randomized design was used with seven replicates per cell. The two factors were curriculum form of repetition and instructional mode of presentation. This yielded four treatment groups: inductive exact (IE), inductive varied (IV), deductive exact (DE), and deductive varied (DV). The statistical models were considered to be completely fixed rather than random.

Procedures

The thirty-two subjects were randomly assigned to the four groups. They spent the first two days in the pretest situation. Part of the first experimental session was devoted to familiarizing the subjects with the teaching machines. Each child was given three lessons a week.

In each lesson the verbal and visual stimuli were presented simultaneously. The subjects were required to respond to approximately four verbal questions in each fifteen minute lesson. Depending on the responses, the machine switched to one of three possible tracks. The first track told the subject he was correct and gave appropriate verbal reinforcement. A second track explained why the subject's

answer was incorrect and gave the correct response. The last track was programmed to explain which choice was correct to those subjects who pressed the "I don't know" button.

The children were given their own schedules and were responsible for being at the testing room in time for their lesson. In all teaching sessions there were at least two experimenters present. These persons served as proctors and machine repairmen when breakage occurred during the sessions. A post content test was administered at the end of the ten lessons.

Tests and Measurements

Since the schools had recently administered WISC intelligence tests to all of the pupils in the sample, these scores were used rather than readministering the test. The mean IQ of the sample was 74.

Mathematical content tests designed specifically for use with EMR's over the concepts which were taught in this phase of the study were not available. Therefore, both the pre and post content test were specifically developed for use in this project. Many of the items of the first phase of the study, however, were modified for use here (Armstrong, 1968a).

The pre (before instruction) content test was administered in two half-hour sessions on two consecutive days. It was designed to provide a rough estimation of the subject's mathematical knowledge in the areas of set terminology, set operations, set relations, number terminology, number relations and number operations. Each item was presented in written form on the test blank and was read aloud two times. A copy of the pretest is included (see Appendix III).

The post (after instruction) content test was also administered in two half-hour sessions. A copy is also included (see Appendix IV). In administering this test, four teaching machines were used to present the alternative choices visually. A slide projector presented the test items in visual form. Again, each item was read twice. The subjects then recorded their answers on Digitek answer sheets which were machine scored. The posttest was designed to measure acquired mathematical knowledge at each of six cognitive levels, in two areas and on three topics. Figure 1 shows the item numbers by category.

Comparability of Treatment Groups

Since the subjects were randomly assigned to each of the four treatment conditions, one would expect that the groups wold be comparatively the same with respect to intellectual level and pre (prior to instruction) mathematical knowledge. Even so, as shown in Table 1, the groups did differ on these two factors. The average IQ's between groups ranged from 70.29 to 77.29. The mathematics content scores ranged from 20.43 to 34.86. The range of the standard deviations was also great between groups. The range of the standard deviations between treatment groups on IQ was 3.04 and on mathematical knowledge was 4.90 (see Table 1).

Cognitive Levels	Sets			Numbers	
	Terminology	Relations	Operations	Terminology	Relations
Knowledge	1,11	18,20	5	25	30
Comprehension	2,12	17,23	15	24	
Application	10,21	16	4,6	26	29
Analysis	3,13		7	27	
Synthesis	22	14	8		
Evaluation	19		9	28	

Figure 1. Post Content Test: Item Classification and Distribution

TABLE I

COMPARABILITY OF TREATMENT GROUPS

Treatment Group	WISC IQ		Pretest	
	Mean	S.D.	Mean	S.D.
Inductive - Varied	70.29	7.02	30.43	6.88
Inductive - Exact	73.57	9.74	35.29	8.69
Deductive - Varied	75.71	10.36	30.57	8.34
Deductive - Exact	77.29	7.52	20.43	8.77

The means and standard deviations herein discussed were based on the 28 subjects, 7 subjects per cell retained for analysis. These subjects were omitted from the final analysis due to sickness which caused them to miss too many lessons to be made-up. They were also absent for the final test. One subject was deleted from one group by random selection to equalize the number in each treatment group.

Statistical Analyses

The data for this phase of the study were analyzed using the same procedures as in phase I (see Armstrong, 1968[a]; pp. 64 and 65). In order to test hypotheses 1 through 3 univariate analyses of variance and covariance were used. To test hypotheses 4 through 12 multivariate analyses of variance and covariance were used.

All of the data analyses were done by computer using "Program Manova" (Clyde *et al.*, 1966). For a more detailed discussion of the statistical procedures used see Armstrong (1968[a]; pp. 62 - 70).

III. RESULTS AND DISCUSSION

In Section I, twelve hypotheses were posed for testing. The procedures used for testing these hypotheses were discussed in Section II. The purpose of this section of the report is to report and discuss the results of these statistical tests of hypotheses.

Tests of Hypothese One through Three: Global Mathematical Learning.

In order to test hypothses one through three, univariate analyses of variance and covariance procedures were used. The results of the analysis of variance on global learning is shown in Table 2. There was no difference found between the exact and varied curriculum repetition groups; therefore, hypothesis one was not rejected (see Table 2). Likewise, there was no difference between the two modes of presentation groups, and thus, hypothesis two was not rejected (see Table 2). The interaction factor between the forms of curriculum repetition and instructional modes of presentation was not significant (see Table 2). Therefore, hypothesis three was not rejected.

Due to the range of difference in the treatment groups on pre (before instruction) mathematical learning and global intelligence (IQ) (see Table 1), covariance analyses were also done. As shown in Tables 3-5, when IQ and prior mathematical learning were used individually or together as covariates in the analyses, the results were the same.

The effects of curriculum and instruction factors on total mathematical learning were similarly not significant in the first phase of the study (see Armstrong, 1968[a]; p. 73). In the first phase of the study, however, the mean squares were much larger in part, one might conjecture, due to the large difference in total sample size. The total sample size of phase I was 228, whereas, the total sample size of this phase of the study was only 28.

In order to test this conjecture $w^2(s)$* were calculated for the two sets of data and are shown in Table 6. The accounted for variance for each of the three factors is quite similar between studies.

In terms of total mathematical learning the amount of accounted for variability due to curriculum, instruction and curriculum by instruction factors is very low (see Table 6). This finding indicates that either these factors are totally ineffective in facilitating the global learning of these pupils or as was the case in phase I (Armstrong, 1968[a]), there are cancelling interactive effects (e.g. treatment A facilitates large amounts of number learning, but low amounts of set learning, while

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* $w^2 = \dfrac{t^2 - 1}{t^2 + (N_1+N_2-1)}$ (Hayes, 1963; p. 327)

TABLE 2

SUMMARY OF ANALYSIS OF

VARIANCE FOR POSTTEST SCORES

Source	Degrees of Freedom	Mean Square	F	P
Curriculum Repetition (Exact vs. varied)	1	8.036	0.607	.444
Instructional Mode of Presentation (Inductive vs. deductive)	1	18.893	1.427	.244
C x I	1	10.321	0.780	.386
Subjects (within cells)	24	13.238		

TABLE 3

SUMMARY OF ANALYSIS OF COVARIANCE

WITH IQ AS COVARIATE

Source	Degrees of Freedom	Mean Square	F	P
Curriculum Repetition (exact vs. varied)	1	3.027	0.253	0.620
Instructional Mode of Presentation (inductive vs. deductive)	1	6.201	0.518	0.479
C x I	1	8.701	0.727	0.403
Subjects (within cells)	23	11.963		

TABLE 4

SUMMARY OF ANALYSIS OF COVARIANCE

WITH PRETEST AS COVARIATE

Source	Degrees of Freedom	Mean Square	F	P
Curriculum Repetition.. (exact vs. varied)	1	2.13	0.18	0.67
Instructional Mode of Presentation (inductive vs. deductive)	1	0.78	0.06	0.79
C x I	1	36.04	3.12	0.09
Subjects (within cells)	23	11.56		

TABLE 5

SUMMARY OF ANALYSIS OF COVARIANCE

WITH BOTH IQ AND PRETEST AS COVARIATES

Source	Degrees of Freedom	Mean Square	F	P
Curriculum Repetition (exact vs. varied)	1	1.07	0.09	0.76
Instructional Mode of Presentation (inductive vs. deductive)	1	0.23	0.02	0.89
C x I	1	25.22	2.23	0.15
Subjects (within cells)	22	11.30		

TABLE 6

OMEGA SQUARED

FOR TOTAL MATHEMATICAL LEARNING

Source of Variation	Omega Squared	
	Phase I (N=228)	Phase II (N=28)
Curriculum	.01	-.03
Instruction	.01	-.04
Curriculum x Instruction	.00	.04

treatment B facilitates large amounts of set learning, but low amounts of number learning, thus, cancelling out any total learning differences between treatment A and treatment B).

Even so, there are some notable differences in the accounted for variability in the total mathematical learning due to curriculum, instruction and curriculum by instruction between the two studies. The curriculum organization factor in the first study did account for more of the total mathematical learning in the first study than did the curriculum repetition factor in the second study (see Table 6). Also, the instructional mode of presentation accounted for more of the total mathematical learning when teachers were used in the first study as opposed to when machines were used in the second study.

Conversely, the amount of variance accounted for due to the curriculum by instruction (C xI) interaction factor was greater in study two than in study one (see Table 6). None of the variability in the total mathematical learning observed in study one was attributable to the C x I interaction factor. In the second study, however, the 4% of the variability in the observed mathematical learning was attributable to the C x I interaction factor.

The difference between the two studies is directly attributable to the difference in curriculum factors examined. In study one, the curriculum factor was organization, area spiral versus topical spiral. In study two, the curriculum factor was repetition, exact versus varied. It would seem, therefore, that the curriculum repetition factor was more prone to vary in facilitating global learning depending on which mode of presentation, inductive or deductive, was used to implement the curriculum. With only 4% (see Table 6) of the total mathematical learning attributable to this factor, however, one would be encouraged to dismiss the relative importance of limiting the use of one form of curriculum repetition, exact or varied, to a single instructional mode of presentation, inductive or deductive.

Test of Hypotheses Four through Six: Mathematical Learning at Various Cognitive Levels

In order to test hypotheses four through six, multivariate and univariate analyses of covariance were used. There was no difference found between the two forms of curriculum repetition, exact and varied, when mathematical learning was assessed at each of six cognitive levels (see Table 7). This finding was consistent when IQ and prior mathematical learning were used both singly and together as covariates (see Tables 8-10). Therefore, hypothesis four was not rejected.

Similarly, there was no difference between the two instructional modes of presentation in facilitating mathematical learning at the various cognitive levels (see Table 7). This finding was also consistent when IQ and prior mathematical learning were used both singly and together as covariates (see Tables 8-10). Therefore, hypothesis five was not rejected.

TABLE 7

SUMMARY OF MULTIVARIATE ANALYSIS

OF VARIANCE ON MATHEMATICAL

LEARNING AT SIX COGNITIVE LEVELS

Source of Variation	Degrees of Freedom Hypothesis	Degrees of Freedom Error	F	P
Curriculum Repetition (Exact vs. varied)	6	19	1.41	.26
Instructional Mode of Presentation (inductive vs. deductive)	6	19	.73	.63
C x I	6	19	2.34	.07

TABLE 8

SUMMARY OF MULTIVARIATE ANALYSIS

OF COVARIANCE ON MATHEMATICAL

LEARNING AT SIX COGNITIVE LEVELS

WITH IQ AS COVARIATE

Source of Variation	Degrees of Freedom Hypothesis	Error	F	P
Curriculum Organization (Exact vs. varied)	6	18	1.24	.33
Instructional Mode of Presentation (inductive vs. deductive)	6	18	.44	.84
C x I	6	18	2.35	.07

TABLE 9

SUMMARY OF MULTIVARIATE ANALYSIS

OF COVARIANCE ON MATHEMATICAL

LEARNING AT SIX COGNITIVE LEVELS

WITH PRETEST AS COVARIATE

Source of Variation	Degrees of Freedom Hypothesis	Degrees of Freedom Error	F	P
Curriculum Repetition (Exact vs. varied)	6	18	1.26	.32
Instructional Mode of Presentation (Inductive vs. deductive)	6	18	.33	.91
C x I	6	18	1.89	.14

TABLE 10

SUMMARY OF MULTIVARIATE ANALYSIS

OF COVARIANCE ON MATHEMATICAL

LEARNING AT SIX COGNITIVE LEVELS

WITH BOTH IQ AND PRETEST AS COVARIATES

Source of Variation	Degrees of Freedom Hypothesis	Degrees of Freedom Error	F	P
Curriculum Repetition (Exact vs. varied)	6	18	1.16	.37
Instructional Mode of Presentation (Inductive vs. deductive)	6	18	.29	.93
C x I	6	18	1.55	.22

The rejection of hypothesis six was questioned. Although the level of significance was only .07 on the curriculum by instruction factor rather than the desired .05 for rejection, the amount of variability in the mathematical learning scores at various cognitive levels attributable to this factor was 5%. Due to this amount of accountable variability, the univariate tests of significance were also examined (see Tables 11-14).

In all cases, with no covariates (see Table 11), with IQ as a covariate (see Table 12), with prior mathematical knowledge as a covariate (see Table 13), and with both IQ and prior mathematical knowledge as covariates (see Table 14), there was a difference found, significant at less than .05 level, in the mathematical comprehension learning among the four treatment groups. The means and standard deviations for mathematical comprehension level learning are shown in Tables 15-18. Table 15 shows the original nonadjusted means for the four groups. Table 16 shows the means adjusted for IQ. Table 17 shows the means adjusted for prior mathematical learning, and Table 18 shows the means adjusted for both IQ and prior mathematical learning. In all cases the inductive - exact group showed the greatest amount of mathematical comprehension followed in order by the deductive - varied, deductive - exact and the inductive - varied group. These findings indicate that when one is concerned with mathematical comprehension level learning the exact form of curriculum repetition was best implemented by the inductive method while the varied form of curriculum repetition was best implemented by the deductive form of presentation.

This finding may be due to the character of the modes of presentation. The deductive mode is primarily a "telling" mode. The deductive lesson was started with an advanced organizer (see Appendix I, deductive lesson) which was a statement of the generalization to be taught during that particular lesson. In the first lesson the generalization was, "a set is a group of things which go together. The things in a set are called <u>members</u> of the set." After the generalization was given a series of examples were used to illustrate this idea. For each set example, the pupils in the deductive mode were told what the set was (e.g. This is a set of dishes.), why these elements were grouped together to form a set (e.g. The things in this set go together because they are all dishes.), and explicitly named for the pupil the members of the set (e.g. The members of this set are two large plates, two small plates, two bowls and two cups.).

In contrast, the inductive mode of presentation was characterized by beginning the lesson with a problem situation. For example, in the first lesson (see Appendix I, inductive approach) the problem situation was set with the statements, "Today, we are going to talk about things you often see grouped together. Let's find out why we group together certain things." The examples in the inductive lesson are the same as those in the deductive lesson. The only difference is the use made of the examples. For each set example, pupils in the inductive mode were not explicitly told what the set was. Rather, the pupils in the inductive mode were expected to determine for themselves

TABLE 11

SUMMARY OF UNIVARIATE ANALYSES

OF VARIANCE ON MATHEMATICAL LEARNING

AT SIX COGNITIVE LEVELS

FOR CURRICULUM REPETITION

BY INSTRUCTIONAL MODE OF PRESENTATION

Source of Variation	Mean Square	Degrees of Freedom Hypothesis	Error	F	P ≤
Curriculum Repetition By Instructional Mode of Presentation:					
Knowledge	2.89	1	24	1.15	.29
Comprehension	11.57	1	24	7.83	.01
Application	2.29	1	24	1.45	.24
Analysis	2.89	1	24	3.12	.09
Synthesis	.04	1	24	.05	.83
Evaluation	1.29	1	24	2.20	.15

TABLE 12

SUMMARY OF UNIVARIATE ANALYSES

OF COVARIANCE ON MATHEMATICAL LEARNING

AT SIX COGNITIVE LEVELS

FOR CURRICULUM REPETITION

BY INSTRUCTIONAL MODE OF PRESENTATION

WITH IQ AS A COVARIATE

Source of Variation	Mean Square	Degrees of Freedom Hypothesis	Degrees of Freedom Error	F	P <
Curriculum Repetition by Instructional Mode of Presentation:					
Knowledge	3.47	1	23	1.58	.22
Comprehension	11.30	1	23	7.43	.01
Application	2.27	1	23	1.38	.25
Analysis	2.67	1	23	2.96	.10
Synthesis	.04	1	23	.06	.82
Evaluation	1.07	1	23	2.33	.14

TABLE 13

SUMMARY OF UNIVARIATE ANALYSES

OF COVARIANCE ON MATHEMATICAL LEARNING

AT SIX COGNITIVE LEVELS

FOR CURRICULUM REPETITION

BY INSTRUCTIONAL MODE OF PRESENTATION

WITH PRETEST AS COVARIATE

Source of Variation	Mean Square	Degrees of Freedom		F	P <
		Hypothesis	Error		
Curriculum Repetition by Instructional Mode of Presentation:					
Knowledge	1.18	1	23	.46	.51
Comprehension	10.89	1	23	7.14	.01
Application	.52	1	23	.33	.57
Analysis	2.26	1	23	2.33	.14
Synthesis	.39	1	23	.60	.45
Evaluation	2.56	1	23	4.80	.04

TABLE 14

SUMMARY OF UNIVARIATE ANALYSES

OF COVARIANCE ON MATHEMATICAL LEARNING

AT SIX COGNITIVE LEVELS

FOR CURRICULUM REPETITION

BY INSTRUCTIONAL MODE OF PRESENTATION

WITH PRETEST AND IQ AS COVARIATES

Source of Variation	Mean Square	Degrees of Freedom Hypothesis	Degrees of Freedom Error	F	P $<$
Curriculum Repetition by Instructional Mode of Presentation:					
Knowledge	2.98	1	22	1.30	.27
Comprehension	9.66	1	22	6.09	.02
Application	.31	1	22	.10	.67
Analysis	1.35	1	22	1.45	.24
Synthesis	.46	1	22	.68	.41
Evaluation	1.52	1	22	3.30	.08

TABLE 15

MEANS AND STANDARD DEVIATIONS

FOR MATHEMATICAL COMPREHENSION

BY TREATMENT GROUP

Treatment Group	Means	Standard Deviations
Inductive - Varied	1.57	.79
Inductive - Exact	3.29	1.11
Deductive - Varied	2.71	1.70
Deductive - Exact	1.86	1.07

TABLE 16

MEANS, ADJUSTED FOR IQ, AND STANDARD DEVIATIONS
FOR MATHEMATICAL COMPREHENSION BY TREATMENT GROUP

Treatment Group	Adjusted Means	Standard Deviations
Inductive - Varied	1.65	.79
Inductive - Exact	3.30	1.11
Deductive - Varied	2.68	1.70
Deductive - Exact	1.80	1.07

TABLE 17

MEANS, ADJUSTED FOR PRIOR MATHEMATICAL KNOWLEDGE,

AND STANDARD DEVIATIONS FOR

MATHEMATICAL COMPREHENSION BY TREATMENT GROUP

Treatment Group	Adjusted Means	Standard Deviations
Inductive - Varied	1.58	.79
Inductive - Exact	3.35	1.11
Deductive - Varied	2.72	1.70
Deductive - Exact	1.77	1.07

TABLE 18

MEANS, ADJUSTED FOR IQ AND PRIOR MATHEMATICAL KNOWLEDGE,

AND STANDARD DEVIATIONS

FOR MATHEMATICAL COMPREHENSION BY TREATMENT GROUP

Treatment Group	Adjusted Means	Standard Deviations
Inductive - Varied	1.54	.79
Inductive - Exact	3.22	1.11
Deductive - Varied	2.68	1.70
Deductive - Exact	1.92	1.07

whether the example shown was a set, and if it was a set, what it was a set of and identify each of the individual members of the set and tell why they were grouped together to form a set.

The reason the inductive mode of presentation better facilitates the understanding of mathematical concepts when the same examples are repeated rather than varied was probably due to certain characteristics of the pupils' thinking processes involved in the acquisition of new ideas.

There seems to be general agreement that the thinking process has step by step quality which aids in the accomodation and assimilation of new ideas (Piaget, 1952, 1953, 1950; Dewey, 1911; Thomson, 1959; Wertheimer, 1945). Piaget (1952) further suggested that one of the crucial steps in the accomodation and assimilation of new ideas was the "structural reorganization" of preliminary concepts which later facilitates the grasping of a new pattern of relationships from which a new idea can evolve.

One might conjecture, then, that the pupils in the inductive - varied mode became involved in only the first step of the thinking process due to the continuing presentation of new and different examples. Pupils in the deductive mode were better able to procede beyond the first step in the thinking process the first time through the examples because much of the information was structurally organized for them in such a way that in essence the first step of their thinking process was completed. They did not have to generalize the particular example as did the pupils in the inductive mode. Therefore, their understanding of the concepts could be further enhanced by exposure to new and different examples during the second half of the lesson.

In contrast, the understanding of the mathematical concepts by the pupils in the inductive mode, since the first step of the thinking process was not completed for them, was more greatly enhanced by the repetition of the same examples. The first time they were exposed to the examples the pupils in the inductive mode structurally reorganized the information so that upon seeing the example again they could procede to the second step of the thinking process. By presenting different examples during the second half of the lesson in the inductive mode the pupil never would get a chance to completely conceptualize the idea. Instead, the pupil in the inductive - varied condition was always involved in only the first stage of the thinking process.

Tests of Hypotheses Seven through Nine: Mathematical Learning in Two Areas.

In order to test hypotheses seven, eight and nine regarding the relative effects of curriculum repetition and instructional mode of presentation factors on the mathematical learning in two areas, sets and numbers, multivariate analyses of variance and covariance procedures were used. There was no evidence found for the rejection of hypothesis seven (see Tables 19-22). None of the F ratios for the curriculum

TABLE 19

SUMMARY OF ANALYSIS OF VARIANCE

ON MATHEMATICAL LEARNING

IN TWO AREAS

Source of Variation	Degrees of Freedom Hypothesis	Degrees of Freedom Error	F	P
Curriculum Repetition (Exact vs. varied)	2	23	2.16	.14
Instructional Mode of Presentation (inductive vs. deductive)	2	23	.71	.50
C x I	2	23	.51	.61

TABLE 20

SUMMARY OF ANALYSIS OF COVARIANCE

ON MATHEMATICAL LEARNING

IN TWO AREAS

WITH IQ AS COVARIATE

Source of Variation	Degrees of Freedom Hypothesis	Degrees of Freedom Error	F	P <
Curriculum Repetition (Exact vs. varied)	2	22	2.13	.14
Instructional Mode of Presentation (Inductive vs. deductive)	2	22	.42	.66
C x I	2	22	.46	.64

TABLE 21

SUMMARY OF ANALYSIS OF COVARIANCE

ON MATHEMATICAL LEARNING

IN TWO AREAS

WITH PRETEST AS COVARIATE

Source of Variation	Degrees of Freedom Hypothesis	Error	F	P<
Curriculum Repetition (Exact vs. varied)	2	22	1.80	.19
Instructional Mode of Presentation (inductive vs. deductive)	2	22	.11	.80
C x I	2	22	1.40	.25

TABLE 22

SUMMARY OF ANALYSIS OF COVARIANCE

ON MATHEMATICAL LEARNING

IN TWO AREAS

WITH BOTH IQ AND PRETEST AS COVARIATES

Source of Variation	Degrees of Freedom Hypothesis	Error	F	P<
Curriculum Repetition (Exact vs. varied)	2	21	1.87	.18
Instructional Mode of Presentation (inductive vs. deductive)	2	21	.14	.87
C x I	2	21	1.03	.37

repetition factor were significant at the acceptable level of .05 (see Tables 19-22). Even so, approximately 4% of the variability of the mathematical learning in these two areas, set and numbers, was attributable to the curriculum repetition factor. Therefore, the univariate analyses of variance and covariance were also examined.

The results of the univariate analyses of variance and covarinace on mathematical learning in the two areas for curriculum repetition are shown in Tables 23-26. There were no differences found between the two repetition groups in the learning of set ideas. There were differences found, however, between the two groups in the learning of number ideas. The percentage of the variability in the number learning attributable to the curriculum repetition factor ranged from 8 to 10.

The original nonadjusted means (see Table 27), the means adjusted for IQ (see Table 28), the means adjusted for prior mathematical knowledge (see Table 29), and the means adjusted for both IQ and prior mathematical knowledge (see Table 30) all favor the exact form of curriculum repetition over the varied form of curriculum repetition.

The mathematical concepts included in the number learning of the program were face value of a numeral, place value of a numeral and equality and non-equality of numbers. These concepts were probably the most difficult for these pupils to grasp. They were all totally new concepts to the pupils. Since the concepts were quite difficult to obtain, the first exposure to the example allowed the pupil to take the first step in the learning process "structural reorganization" while the second exposure to the example allowed the pupil to take the second step in the learning precess, that of actually conceptualizing the concepts.

Earlier, in examining the comprehension learning of the EMR pupils in the study, however, it was found that the deductive - varied mode better facilitated the learning of these pupils than did the deductive - exact. Here, however, in terms of overall number learning there was a reversal. In terms of total number learning across cognitive levels the mathematical learning of the deductive and inductive exact groups was greater than the deductive and inductive varied groups. An examination of Figure 2 illustrates that the earlier finding was stable with respect to number learning. Number comprehension (understanding of number concepts) was best facilitated by the inductive - exact and the deductive - varied modes. Therefore, the result is consistent across comprehension learning (see Table 2).

The reversal from the deductive - varied to the deductive - exact in number learning, exclusive of comprehension level learning, was primarily due to knowledge, application, and evaluation level learning (see Figure 2) of numbers. The previous conjecture regarding the explanation of the comprehension level learning of mathematical concepts in general must thus be expanded to explain the reversal for knowledge, application and evaluation level learning of numbers.

The posttest items used to assess number learning at the knowledge, application and evaluation levels were numbers 25-30 (see

TABLE 23

SUMMARY OF UNIVARIATE ANALYSES

OF VARIANCE ON MATHEMATICAL LEARNING

IN TWO AREAS FOR CURRICULUM REPETITION

Source of Variation	Mean Square	Degrees of Freedom Hypothesis	Degrees of Freedom Error	F	P<
Curriculum Repetition:					
Sets	.04	1	24	.01	.95
Numbers	9.14	1	24	4.41	.05

TABLE 24

SUMMARY OF UNIVARIATE ANALYSES

OF COVARIANCE ON MATHEMATICAL LEARNING

IN TWO AREAS FOR CURRICULUM REPETITION

WITH IQ AS COVARIATE

Source of Variation	Mean Square	Degrees of Freedom Hypothesis	Error	F	P <
Curriculum Repetition:					
Sets	.70	1	23	.09	.77
Numbers	8.70	1	23	4.03	.06

TABLE 25

SUMMARY OF UNIVARIATE ANALYSES

OF COVARIANCE ON MATHEMATICAL LEARNING

IN TWO AREAS

FOR CURRICULUM REPETITION

WITH PRETEST AS COVARIATE

Source of Variation	Mean Square	Degrees of Freedom Hypothesis	Error	F	P <
Curriculum Repetition:					
Sets	.40	1	23	.04	.84
Numbers	7.40	1	23	3.59	.07

TABLE 26

SUMMARY OF UNIVARIATE ANALYSES

OF COVARIANCE OF MATHEMATICAL LEARNING

IN TWO AREAS

FOR CURRICULUM REPETITION

WITH IQ AND PRETEST AS COVARIATES

Source of Variation	Mean Square	Degrees of Freedom Hypothesis	Error	F	p <
Curriculum Repetition:					
Sets	1.26	1	22	.16	.70
Numbers	7.49	1	22	3.48	.08

TABLE 27

MEANS AND STANDARD DEVIATIONS

FOR MATHEMATICAL LEARNING OF NUMBERS

BY FORM OF CURRICULUM REPETITION

Form of Curriculum Repetition	Means	Standard Deviations
Exact	4.22	1.23
Varied	3.07	1.62

TABLE 28

MEANS, ADJUSTED FOR IQ,

AND STANDARD DEVIATIONS

FOR MATHEMATICAL LEARNING OF NUMBERS

BY FORM OF CURRICULUM REPETITION

Form of Curriculum Repetition	Adjusted Means	Standard Deviations
Exact	4.20	1.23
Varied	3.09	1.62

TABLE 29

MEANS, ADJUSTED FOR PRETEST,

AND STANDARD DEVIATIONS

FOR MATHEMATICAL LEARNING OF NUMBERS

BY FORM OF CURRICULUM REPETITION

Form of Curriculum Repetition	Adjusted Means	Standard Deviations
Exact	4.51	1.23
Varied	3.12	1.62

TABLE 30

MEANS, ADJUSTED FOR PRETEST AND IQ,

AND STANDARD DEVIATIONS

FOR MATHEMATICAL LEARNING OF NUMBERS

BY FORM OF CURRICULUM REPETITION

Form of Curriculum Repetition	Adjusted Means	Standard Deviations
Exact	4.19	1.23
Varied	3.00	1.62

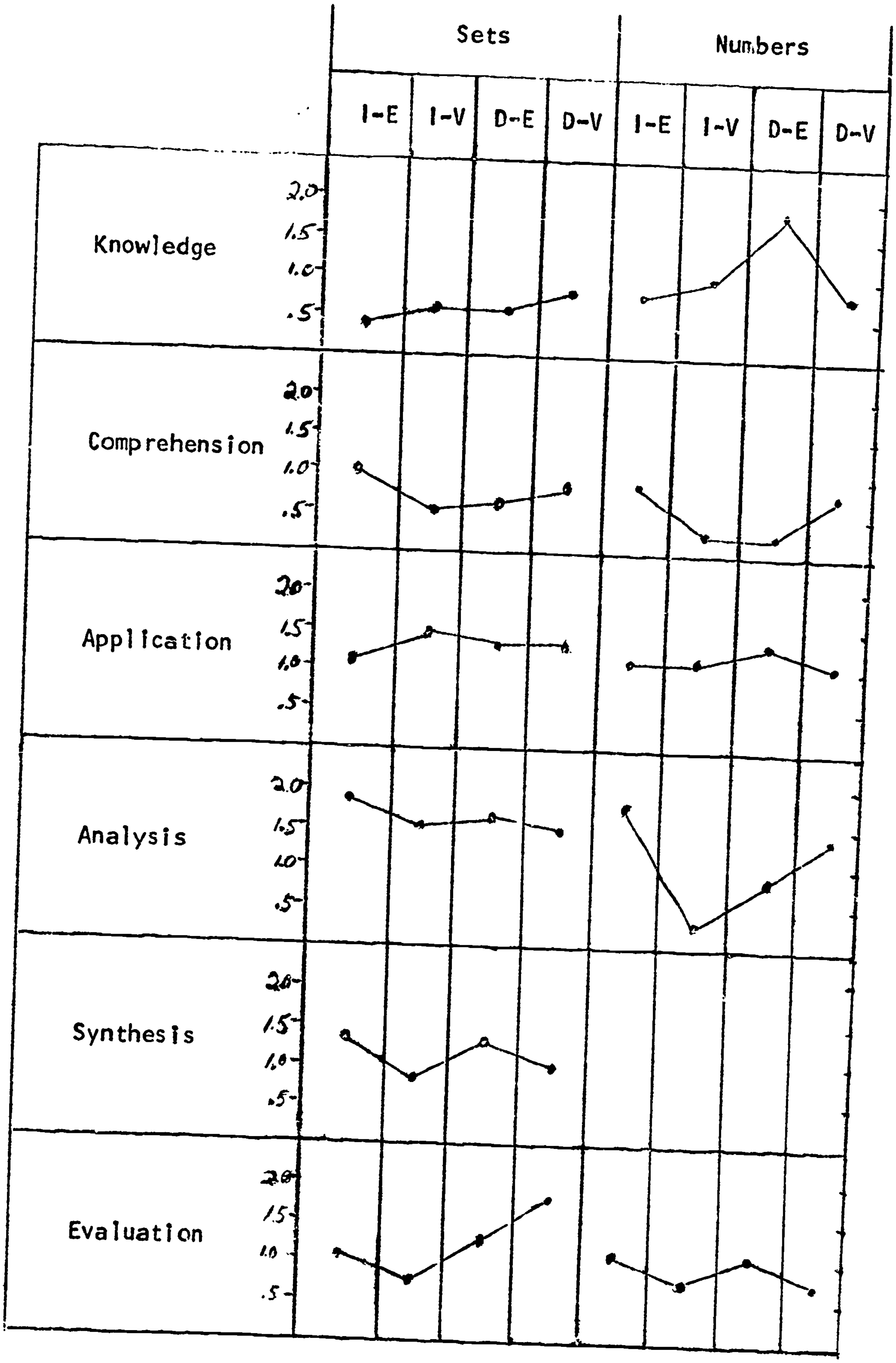

Figure 2. Means: Directional Plots by Treatment Group for Various Types of Learning

Figure 1). These items are shown in Appendix IV. The content of the items was place and face value of numerals and the equality and non-equality of numbers. The correct completion of the knowledge level items required the recall of the names for the three places studied and recall of the "is equal to" symbol. The successful completion of the application level number items was dependent on the pupil's ability to 1) determine the correct way to write the numeral which told the number represented by a set of blocks and 2) determine which pair of numbers were not equal among four sets of two numbers (see Appendix IV, items 26 and 29). To successfully complete the evaluation level number item the pupils had to evaluate the number of elements in each set amoung four sets to determine which number did not represent the numeral 10. The comprehension level item in contrast required the child to really understand in depth the place value-face value concepts. The successful completion of the comprehension level number item required determination of what a particular single diget numeral meant when it was positioned in one three places, units, tens or hundreds.

The successful learning of number concepts at the knowledge, application and evaluation levels required the exact repetition of examples. Seemingly, the learning of number concepts, with the exception of number learning at the comprehension level, required the structural reorganization of the relationships in a constant setting in order for the pupils to learn. The fact that this is consistent irrespective of the mode of presentation used is puzzling, in light of the previous finding regarding comprehension level learning of all mathematical concepts. This finding would suggest that there is wide variability in the general thinking precesses of EMR pupils depending upon the content being studied, the level of learning taking place, and the specific characteristics of the curricular and instructional factors to which he is being subjected.

More specifically, the mathematical learning of number concepts at the knowledge, application and evaluation levels was best facilitated by an exact form of curriculum repetition. Comprehension level learning of all mathematical concepts was best facilitated by either an inductive mode of presentation coupled with the exact repetition of examples or a deductive mode of presentation coupled with a varied form of repetition of examples.

Due to the internal interactive effects found, hypothesis seven _in toto_ was not rejected.

No evidence was found for the rejection of hypothesis eight regarding the relative effects of the instructional modes of presentation on learning in two areas (see Tables 19-22). None of the overall F ratios reached the desired level of .05. Also, essentially none of the variability of the mathematical learning in the two areas was directly attributable to the instructional mode of presentation factor.

Similarly, no evidence was found for the rejection of hypothesis nine regarding the C x I interaction effects on mathematical learning in the two areas (see Tables 19-22). Again, none of the variability of the mathematical learning in the two areas was directly attributable to

the curriculum by instruction factor.

Tests of Hypotheses Ten through Twelve: Mathematical Learning on Three Topics

In order to test hypotheses ten through twelve, multivariate analyses of variance and covariance procedures were used. There was no evidence found for the rejection of any of the three hypotheses (see Tables 31-34). The largest F ratios were found on the C x I factors in the four analyses. In any case, only 4% of the variability in the mathematical learning of topics was attributable to the C x I factor. This level of accounted for variability occurred in only one instance (see Table 33) out of four (see Tables 31-34). Therefore, hypotheses ten, eleven and twelve were not rejected.

TABLE 31

SUMMARY OF ANALYSIS OF VARIANCE

ON MATHEMATICAL LEARNING

ON THREE TOPICS

Source of Variation	Degrees of Freedom Hypothesis	Error	F	P
Curriculum Repetition (Exact vs. varied)	3	22	.92	.45
Instructional Mode of Presentation (inductive vs. deductive)	3	22	.98	.42
C x I	3	22	1.26	.31

TABLE 32

SUMMARY OF ANALYSIS OF COVARIANCE

ON MATHEMATICAL LEARNING

ON THREE TOPICS

WITH IQ AS COVARIATE

Source of Variation	Degrees of Freedom Hypothesis	Error	F	P
Curriculum Repetition (Exact vs. varied)	3	21	.71	.56
Instructional Mode of Presentation (Inductive vs. deductive)	3	21	.79	.52
C x I	3	21	1.21	.33

TABLE 33

SUMMARY OF ANALYSIS OF COVARIANCE

ON MATHEMATICAL LEARNING

ON THREE TOPICS

WITH PRETEST AS COVARIATE

Source of Variation	Degrees of Freedom Hypothesis	Error	F	P
Curriculum Repetition (Exact vs. varied)	3	21	.67	.58
Instructional Mode of Presentation (inductive vs. deductive)	3	21	.84	.49
C x I	3	21	2.10	.13

TABLE 34

SUMMARY OF ANALYSIS OF COVARIANCE

ON MATHEMATICAL LEARNING

ON THREE TOPICS

WITH BOTH IQ AND PRETEST AS COVARIATES

Source of Variation	Degrees of Freedom Hypothesis	Error	F	P
Curriculum Repetition (Exact vs. varied)	3	20	.58	.64
Instructional Mode of Presentation (inductive vs. deductive)	3	20	.86	.48
C x I	3	20	1.60	.23

IV. SUMMARY AND CONCLUSIONS

The purpose, methodology, and results of the investigation are briefly summarized in the first part of this section. The discussion of the second part of this section is devoted to limitations, conclusions and implications.

Restatement of the Problem

The purpose of this study was to determine the relative effects of two instructional modes of presentation and two curriculum forms of repetition on mathematical learning at six cognitive levels, in two areas and on three topics. The instructional modes of presentation compared were inductive and deductive. The curriculum forms of repetition compared were exact and varied.

Experimental Program

Four different programs were developed for each of ten lessons which covered the following concepts: set terminology - definition, membership and cardinality, set relations - subset and 1-1 correspondence, set operation - unioning, number terminology - place value and face value, number relations - equality and non-equality. The four programs differed only in two factors. The two factors were curriculum form of repetition, exact versus varied, and instructional mode of presentation, inductive versus deductive.

The four different treatment conditions, inductive - exact (I-E), inductive - varied (I-V), deductive - exact (D-E), and deductive - varied (D-V), were derived from two scripts written for each lesson. The (I-V) and (D-V) conditions used the scripts in their original form. The (I-E) and (I-V) conditions used a modified form wherein the first half of the script was exactly repeated and the second half omitted. Thus, each lesson consisted of two parts. In the exact repetition conditions, the subjects viewed the $7\frac{1}{2}$ minute sequence presented either inductively or deductively twice. In the varied repetition conditions, during the second $7\frac{1}{2}$ minutes, the subjects viewed different examples from those shown in the first half of the lesson. Each lesson, then, for each pupil was approximately 15 minutes in length with from 40-60 slide changes in each lesson.

Color slides were produced from original artwork transparencies of everyday objects or illustrative number examples. To provide some personal identification source for the subjects, a character, Mr. Mathematics, was created.

The 40 scripts, 10 lessons of four different treatment conditions, were all recorded by a single person experienced in educational radio and television announcing and broadcasting. The original tapes were then edited and programmed with inaudible "bleeps" to trigger slide changes at the appropriate times in the program sequence. The programs

were implemented with four Ken Cook Company Mark VII Machines.

Experimental Procedures

Thirty-two educable mentally retarded pupils with mean IQ of 74 and average C.A. of 12.6 were selected for participation in the study. The subjects were randomly assigned to each of the four treatment conditions. Three subjects were deleted from the final sample due to illness. Another subject was randomly deleted to equalize the cells thus leaving a total N = 28.

A pretest of mathematical knowledge was given prior to instruction, and a posttest of mathematical learning at six cognitive levels, in two areas and on three topics was administered immediately after the completion of instruction. These tests were developed specifically for use in this project.

The experimental design was a 2x2 completely crossed and randomized design with 7 replicates per cell. The models were considered to be fixed rather than random.

Multivariate analyses of variance and covariance procedures were utilized to analyze the data involving subtests. Univariate analyses were used when total or composite test score data were analyzed. The level of acceptable probability was set at .05. Due to the small sample size, however, when the accounted for variance reached 5% or greater univariate post hoc analyses were also examined.

Results

Twelve hypotheses were proposed (see pp. 2 and 3) and tested (see pp. 10-50). The three major factors investigated, curriculum repetition, instructional mode of presentation, and curriculum repetition by instructional mode of presentation are discussed separately.

Curriculum Repetition

The form of curriculum repetition, exact or varied, differentially affected the mathematical learning of EMR's in two areas. Number learning at the knowledge, application and evaluation levels was better facilitated by the exact form of curriculum repetition than by the varied form of curriculum repetition irrespective of the mode of presentation used for implementation. Due to the interactive effects, when global learning was examined in terms of the curriculum repetition factor, no differences were found.

This finding, regarding the general curriculum factor was consistent with the first study (see Armstrong, 1968[a]; p. 134). In the first study, the curriculum factor examined was organization, topical spiral versus area spiral, whereas the curriculum factor in the second study was repetition, varied or exact. The curriculum organization factor was also found to significantly affect the learning of number

concepts at the knowledge, application and evaluation levels of learning. Seemingly, therefore, even though the curriculum factors examined were different between the two studies, both affected the same types of mathematical learning at the same cognitive levels of learning.

These findings indicate that both the form of organization and the form of curriculum repetition are critical factors in the design of programs which are concerned with number learning at the knowledge, application and evaluation levels for both EMR's and above average normals. Furthermore, this finding reiterates the need for seperating the curriculum and the instruction factors in the design of educational research (see Armstrong, 1967). Too often, research has focused on the effects of the instructional methods variables with little or no concern for either the control of or examination of the curriculum factor. Certainly, these findings would suggest that the curriculum factor should be considered more seriously in the design of future studies and in the evaluation of textbook materials for classroom use.

Instructional Mode of Presentation

The only difference found due to the instructional modes of presentation, inductive or deductive, in the first study (see Armstong, 1968[a]; p. 134) was on the mathematical learning operations. The inductive mode was found to better facilitate the learning of operations than did the deductive mode (see Armstrong, 1968[a]; p. 97). It was surprising to find so few differences attributable to the instruction factor (Armstrong, 1968[a]). Therefore, this study was undertaken to verify the findings of the Armstrong (1968[a]) study.

As previously stated, one conjecture was that the absence of differences was due to lack of complete control over the teacher variable. Thus, this study examined the same instructional modes of presentation using teaching machines rather than teachers. Thus, the verbalization of the two modes was completely controlled.

The results of this study, however, indicated that the findings in the first study for the instruction factor (Armstrong, 1968[a]) were undoubtedly valid. The findings of this study also indicate that there were no differences in any of the types or levels of mathematical learning which were directly attributable to the instruction factor. In contrast to the first study (Armstrong, 1968[a]), however, differences in the learning of operations attributable to the instruction factor were not found. The findings of other investigators upon examination of the inductive and deductive modes of instruction have also been varied. These findings are reviewed in detail by Armstrong (1968; pp. 19-22). The conclusion drawn from this review was that the difference in findings among the various studies on the instructional factor were due in part to the chronological age or Piagetian stage of the pupils involved in the studies (see Armstrong, 1968[a]; p. 20). If this was a valid conclusion, then the difference in the results between the present study and the Armstrong (1968[a]) study may be due to the different types of pupils involved in the two studies. Although the mean

chronological age of the subjects in the two studies were approximately the same, the mental ages were quite divergent. The average mental age of the EMR sample was 9.32 while the average mental age of the Armstrong (1968[a]) above average normal sample was 14.04.

Research which has been completed with pupils of similar chronological age but contrasting mental age on the Piagetian stages (Flavell, 1963), indicated that the stages did occur in the same order, but there was a lag attributable to mental age (Quick, 1966). Pupils with lower mental ages were found to lag behind pupils of the same chronological age but with higher mental ages in reaching the various Piagetian stages. Even so, they did pass through the stages in the same order.

The mathematical learning of the EMR pupils who would, due to their mental age lag, be in the concrete operations stage seemed to be unaffected by the mode of presentation used. The mathematical learning of operations, however, of the above average normals (Armstrong, 1968[a]) who would be in the formal operations stage was affected by the mode of presentation used. The findings of these studies, however, do not complement an earlier conjecture made by Ausubel (1963) regarding the relationship between Piagetian stages and the instructional method. Ausubel (1963), although not referring to EMR pupils per se suggested that pupils in the beginning Piagetian stages would profit more from the inductive mode of presentation while pupils in the later stages would profit more or equally as well from the deductive mode of presentation.

Certainly, the interrelationships among pupil mental age, chronological age, methods of instruction and Piagetian developmental stages need to be explored further. The divergent results both in these studies and in previous studies (see Armstrong, 1968[a]; pp. 19-22) make further exploration of the inductive and deductive modes of presentation in conjunction with various learner variables a badly needed next step in the area of mathematical learning.

Curriculum by Instruction

In contrast to the first study (Armstrong, 1968[a]), wherein there was no mathematical learning variability directly attributable to the curriculum by instruction (C x I) factor, the results of this study showed 4% of the total mathematical learning attributable to the C x I factor, 5% of the mathematical learning at various cognitive levels attributable to the C x I factor, and 21% of the variability in comprehension level learning attributable to the C x I factor.

These findings point up a basic difference between the two curriculum factors studied. The form of curriculum organization used was equally effective in facilitating mathematical learning irrespective of the mode of presentation used to implement the program. In contrast, however, the form of curriculum repetition used was not unaffected by the mode of presentation used to implement it.

The exact form of curriculum repetition better facilitated learning at the comprehension level when implemented by the inductive mode. The

varied form of curriculum repetition, however, better facilitated mathematical learning at the comprehension level when implemented by the deductive mode.

This finding may best be explained in terms of the step by step nature of the child's thinking processes. Since the deductive mode structurally organizes the learning situation for the child, that is, takes the first step of the thinking process for him, the comprehension learning of the pupils in the deductive mode was not as greatly facilitated by the repetition as by the variation of examples. In contrast, since the inductive mode does not structurally organize the examples for the child, the comprehension level learning of the pupils in the inductive mode was more greatly facilitated by the repetition of examples than by the variation of examples.

Limitations

There were three major limitation: the size of the sample, the limited capabilities of the teaching machine programs, and the machine breakage factor. Each limitation is discussed separately.

One major limitation of the study was the samll number of subjects in the total sample. Since teaching machines were used rather than teachers, only four subjects at a time could be involved in the experimental program. In contrast, by using teachers larger numbers of pupils can be exposed to the treatment conditions at a time, thus, allowing for larger sample sizes.

The second major limitation was the limited capability of a machine in the instructional setting. For example, in programming the inductive mode it was impossible to vary the responses depending upon the pupil's responses to the questions. Although the teaching machines used provided three different tracks for "teacher" response to the child's response to certain questions, the pupil only could respond with one of three apriori determined responses. When actually implementing via the inductive mode in the classroom, the pupils are able to respond to the questions in many different ways and the teacher can modify her original question by the way in which a particular child responds. This is not possible when using the teaching machine. Consequently, many of the inductive mode presentations were composed of far too many rhetorical questions.

The third major limitation was the difficulty encountered in keeping the machines running. The machines, being quite intricate in nature, were constantly breaking down. Many times this came in the middle of a lesson and thus the sequence of the pupil's instructional session was broken. Also, at times the tapes slipped so that the tracks were transposed resulting in incorrect "teacher" responses for pupil answers to questions. When this happened, proctors were on hand to correct the mistake. Many times, however, it was too late for the pupil who had already been told that his correct response was wrong, thus, resulting in extreme confusion for the subject involved.

The results of this study together with the results of the previous study (Armstrong, 1968[a]), irrespective of the several limitations previously discussed, suggest certain conclusions and implications for future research and practice.

Conclusions and Implications

Due to the length of discussion of the results in the previous section, the conclusions and implications follow in summary form.

Conclusions

1. Number learning at the knowledge, application and evaluation levels is better facilitated by an area spiral form of curriculum organization (Armstrong, 1968[a]) and an exact form of curriculum repetition.

2. Mathematical learning of operations is better facilitated by an inductive rather than a deductive mode of presentation for pupils of normal mental age (Armstrong, 1968[a]), but equally facilitated by an inductive or deductive mode for EMR pupils.

3. The form of curriculum organization used, area or topical, is equally effective in facilitating mathematical learning irrespective of the mode of presentation, inductive or deductive, used to implement the program (Armstrong, 1968[a]). The exact form of curriculum repetition better facilitates mathematical learning at the comprehension level when implemented by the inductive mode of presentation, while the varied form of curriculum repetition better facilitates mathematical learning at the comprehension level when implemented by the deductive mode of presentation.

Implications

The replication of the first study (Armstrong, 1968[a]) has provided added insight into the curriculum, instruction, and the curriculum by instruction factors herein investigated. The results of these studies point up certain needed next steps in the area of mathematics learning research. Two of the more important areas of concern should be: 1) more concentrated research on the curriculum factor as apart from the instruction, teacher and learner factors (Armstrong, 1967), and 2) the investigation of the interrelationships among pupil mental age, chronological age, the inductive and deductive modes, and Piagetian developmental stages.

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APPENDIX I

Lesson Number One

Lesson Number One: Introduction to Sets
Set Terminology
Inductive Approach

1:1 I want you to meet Mr. Mathematics. He's going to help me show you some things about mathematics which you may not have learned before. As you can see, he has three three baskets of mathematics: numbers, shapes, and sets. In Mr. Mathematics' first basket are numbers. You probably know about this kind of mathematics. Sometimes we call this kind arithmetic. There are two other kinds of mathematics you may not know about. In Mr. Mathematics' second basket are shapes. In Mr. Mathematics' third basket are sets. We'll be exploring the mathematics in each of Mr. Mathematics' baskets. We'll be studying numbers, shapes, and sets. Today we are going to talk about things you often see grouped together. Let's find out why we group together certain things.

2:2 We often see dishes on a breakfast table. Are these dishes alike in any way? We can see that all the dishes are the same color but is color the best reason for putting them together in a group? Are the plates dishes? Are the cups dishes? Are the bowls dishes? What is the best reason for putting the plates, the cups and the bowls together in a group?

3:6 You may have seen these before on the playground. What are they? Why do we place the two swings together? Do we use the two swings for the same purpose?

4:7 Here is one of the swings. How do we use this swing? Do we swing in the swing?

5:8 Here is the group of swings. Do we use <u>both</u> swings for the same purpose?

6:9 Look at this group of animals. What are the animals in this group? That's right. There is a dog, a cat and a bird in this group. Why have we placed these animals together in a group? How are these animals alike? Do they all have four legs? No, because the bird does not have four legs. So, the number of legs they have does not make them alike. Could each of these animals be kept as a pet? Let's check each one and see.

7:10 Here is the dog. Could a dog be kept as a pet? If you say yes, you were right. A dog can be kept as a pet?.

8:11 What about a cat? Could a cat be kept as a pet? Again if you said yes, you were right. A cat can be kept as a pet.

9:12 Think about this bird. Could a bird like this be kept as a pet? Yes We could keep a bird like this as a pet

10:9 Are all of the animals in this set pets? Is the dog a pet? Is the cat a pet? Is the bird a pet? Is that a good reason for putting all of these animals together in a group?

11:55 Look at the objects in this picture. What are they? Could the swing be a member of the group of pets?
12:47 If your answer is yes, push the green button If your answer is no, push the yellow button If you do not know the answer push the red button

13:50 Remember the question: Is the swing a member of the group of pets?

14:9 GREEN TRACK: No you are wrong The swing cannot be a member of the group of pets because a swing is not a pet. Every member in the group of pets is a pet.

YELLOW TRACK: Yes, that's right. The swing cannot be a member of the group of pets because a swing is not a pet. You were right. Every member in the group of pets is a pet.

RED TRACK The answer is no. The swing cannot be a member of the group of pets because a swing is not a pet. Every member in the group of pets is a pet.

15:25 Here is a group you often see. What are the members of this group? Why are the girls members of this group? Why are the boys members of this group? Are the boys children? Are the girls children? Do we put these girls and boys together in a group because they are all children?

16:33 Look at the members of this group. What are they? Why do we put these books together in a group? Are the books all the same color? No, they aren't the same color. So, that isn't the reason they are placed in a group.

17:34 Is the red book one of the members of the group? How is the red book similar to the other books?

18:35 Is the green book similar to the other objects on the table?

19:33 Let's take a close look. Do you see the red book? Do you see the green book? How are all these things on the table top similar?

20:56 Is this baby similar to the books on the table? Is the baby a book?

21:49 If your answer is yes press the green button. Push the yellow button if your answer is no. Push the red button if you do not know the answer.

22 51 Remember the question? Is the baby like the books on the table? Is the baby a book?

23:33 TRACK GREEN: You are wrong. The baby is not a member of the set of books. The baby cannot be a member of the set of books because the baby is not a book.

TRACK YELLOW: You are right. The baby is not a member of the set of books. You did very well. You know that the only things which can be members of the set of books are books.

TRACK RED: The answer is no. The baby is not a member of the set of books. The baby cannot be a member of the set of books because the baby is not a book. The only things which can be members of the set of books are books.

24:46 What do we have here in this group? How are all the balls similar? It's not the colors that make them alike. What is it that makes them alike?

25:47 Is this ball one of the balls in the group of balls? Why can we say that this ball can be placed with the other balls.

26:3 Here are many golf clubs. How are they similar? Why do you think we can place them all in one group? There are two types of golf clubs aren't there? Couldn't we place all the golf clubs made of wood together? Couldn't we place all the other golf clubs made of iron together? Yet, we can also put all the golf clubs together -- but why is it that we can put them together?

27:16 What are some of the foods you see here? Are these food items alike?

28:17 Is the apple a fruit? Is it one of the food items in the original group of food?

29:18 Is the orange similar to the other foods from the bunch of foods? Is the orange a fruit?

30:20 How about grapes? Is a grape a fruit? Is this food item like the others in the group?

31:16 Let's think for a moment? Here are all the food items. How are they alike? Is each food item a fruit?

32:57 Is this cat a member of the set of fruit?

33:49 If your answer is yes, press the green button. If your answer is no, press the yellow button. If you do not know the answer, press the red button.

34:59 Remember the question: Is this cat a member of the set of fruit?

35:16 GREEN TRACK: No you are wrong. The cat cannot be a member of the set of fruit because a cat is not a fruit. Remember, the only members of the set of fruit are fruits.

YELLOW TRACK: Yes that's right. The cat cannot be a member of the set of fruits because a cat is not a fruit. You were right. Every member of the set of fruit is a fruit.

RED TRACK: The answer is no. The cat cannot be a member of the set of fruit because the cat is not a fruit. Every member of the set of fruits is a fruit.

36:30 Here's a fellow carrying something. What is he carrying? How are all these objects alike? Do you think they are alike? All the balloons are different in color but what is a reason for putting them all in the one group? Is the red balloon a balloon? Is the yellow balloon a balloon? How about the green balloons? Is the green balloon a balloon?

37:38 Have you seen these before? What are they? Perhaps sometimes you are sent to the store to do some shopping for mother. If you were, you probably had a set of coins in your pocket? Are these different kinds of coins? What are they? What are the different kinds of coins that you see?

38:39 Is the nickel a coin? Is it similar to the other members of the group of money? Why is a nickel like the other coins?

39:40 Here's a dime. Is it a coin? Is it similar to the other coins? Are these reasons for placing these things together?

40:7_/ This is a set of horses.

41:60 Is this cat a member of the set of horses?

42:49 If your answer is yes press the green button. If your answer is no press the yellow button. If you do not know press the red button.

43:59 Remember the question, is this cat a member of the set of horses?

44:237 GREEN TRACK: No you are wrong. The cat cannot be a member of the set of horses because the cat is not a horse. Remember the only members of the set of horses are horses.

YELLOW TRACK: Yes that's right. The cat cannot be a member of the set of horses because the cat is not a horse. You were right every member of the set of horses is a horse.

RED TRACK: The answer is no. The cat cannot be a member of the set of horses because the cat is not a horse. Every member of the set of horses is a horse.

45:43 Here's another group of objects. What are they?

46:53 If you had these pencils you could draw and draw and draw.

47:43 How are all these pencils similar? We can see that the color of the pencils are all different, so that can't be a reason. Why, then, can we group these pencils together?

48·44 What color pencil do we have here? Is it one of the pencils we saw in the group of pencils?

49:1 Today we have talked about groups of things. We found out that similar or objects that are alike can be grouped together only if they have something in c ommon. For example, we talked about the set of balloons. All the members belonged to the set because each balloon was a balloon. An airplane is not a balloon so we couldn't place it in the same set. Can you think of a set? I'll see you tomorrow when we'll do some more exploring in Mr. Mathematics' basket of sets. Good-bye for now.

Lesson One: Introduction to Sets
Set Terminology
Deductive Approach

1:1 Hello! I want you to meet Mr. Mathematics. He's going to help me show you some things about mathematics which you may not have learned before. As you can see, he has three baskets of different kinds of mathematics. You probably know about the kind of mathematics that uses numbers. Sometimes we call this kind, arithmetic. In Mr. Mathematics' first basket are numbers. There are two other kinds of mathematics in his baskets. Mr. Mathematics' second basket is filled with shapes. Mr. Mathematics' third basket is filled with sets. We'll be exploring the mathematics in each of Mr. Mathematics' baskets. We'll be studying sets, shapes, and numbers. I think you'll like working with sets, shapes and numbers. Today we are going to talk about some of the sets which you see every day. But what is a set? A set is a group of things which go together. We call these things in a set, members of the set.

2:2 Here is a set you may have seen on the breakfast table this morning. This is a set of dishes. The things in this set go together because they are all dishes. Every member of this set is a dish. The members of this set are two large plates, two small plates, two bowls and two cups. This is a set of dishes.

3:6 Here is another set you may have seen recently. You may have seen a set like this on the playground. Have you ever swumg in a swing? This is a set of swings. Every member of this set is a swing. You can swing in every member of this set.

4:7 This is one member of the set of wings. You can swing in this set member.

5:6 Every member of this set is a swing. You can swing in every member of this set.

6:54 Have you ever had a pet?

7:9 This set of pets has in it a dog, a cat, and a bird. The pets in this set are each a different color. The dog is brown. The cat is white and the bird is green. But the reason these animals are members of the set is because they are all pets.

8:10 The dog is a pet. Therefore, he is a member of the set of pets.

9:11 The cat is also a member of the set because he is a pet.

10:12 The bird is also a member of the set because he is a pet.

11:9 The dog, the cat, and the bird make up the set of pets. Every member of the set of pets is a pet.

12:55 Is this swing a member of the set of pets?

13:49 If your answer is yes, push the green button. If your answer is no, push the yellow button. If you do not know the answer, push the red button.

14:50 Remember the question: Is this swing a member of the set of pets?

15:9 GREEN TRACK: No, you are wrong. The swing cannot be a member of the set of pets because the swing is not a pet. Every member of the set of pets is a pet.

YELLOW TRACK: Yes, that's right. The swing cannot be a member of the set of pets because a swing is not a pet. You were right. Every member of the set of pets is a pet.

RED TRACK: The answer is no. The swing cannot be a member of the set of pets because a swing is not a pet. Every member of the set of pets is a pet.

16:25 Another set which you see every day is the set of boys. There are sets of boys in your classroom, aren't there? This set of boys has five members. Each boy has a different colored shirt. All of these boys are members of the set. They are all members of this set because they are all boys. A girl could not be a member of this set. Only boys can be members of this set. This is a set of boys.

17:33 You probably saw this set today in your classroom. This is a set of books. Each of these books is a different color, but you can read every member of this set.

18:34 You can read the red book. The red book is a member of the set. Any book could be a member of this set.

19:35 This green book is also a member of the set of books. You can read this green book.

20:33 All of the books on this table are members of the set. This is a set of books.

21:56 Is this baby a member of the set of books?

22:49 Push the green button if your answer is yes. Push the yellow button if your answer is no. Push the red button if you do not know the answer.

23:51 Remember the question? I want to know, is this baby a member of the set of books?

24:33 <u>GREEN TRACK:</u> You are wrong. The baby is not a member of the set of books. The baby cannot be a member of the set of books because the baby is not a book. The only things which can be members of the set of books are books.

<u>YELLOW TRACK:</u> You are right. The baby is not a member of the set of books. You did very well. You know that the only things which can be members of the set of books are books.

<u>RED TRACK:</u> The answer is no. The baby is not a member of the set of books. The baby cannot be a member of the set of books because the baby is not a book. The only things which can be members of the set of books are books.

25:46 The set we are going to talk about now is the set of balls. Each ball is a different color. You can bounce every member of this set. You can roll every member of this set, and you can throw every member of this set.

26:47 Here is one member of the set of balls. You can bounce this ball. You can roll this ball and you can throw this ball. This ball is a member of the set of balls.

27:3 Here is a set of golf clubs. Have you ever played golf? I play sometimes. Sometimes I watch other people play golf on television. You could use this set to play golf. The members of this set go together because they are all used to play golf. They are all golf clubs. Some of the members of the set are called irons, because they are made out of iron. Some of the members of this set are called woods, because they are made out of wood. But all of the woods and all of the irons are members of the set because they are all golf clubs. All of these golf clubs together in a group make up the set of golf clubs.

28:16 This set will make you hungry. You may have seen this set at lunch time. This set of fruits has as its members an apple, an orange, a banana, and a bunch of grapes. You can eat every member of this set.

29:17 You can eat an apple. The apple is a member of

the set of fruits.

30:18 You can eat an orange. The orange is a member of the set of fruits.

31:19 You can eat a banana. The banana is a member of the set of fruits.

32:20 You can eat grapes. This bunch of grapes is also a member of the set of fruits.

33:16 The apple, the orange, the banana, and the bunch of grapes are all members of the set because they are all pieces of fruit.

34:57 Is this cat a member of the set of fruit?

35:49 If your answer is yes, press the green button. If your answer is no, press the yellow button. If you do not know, press the red button.

36:59 Remember the question: Is this cat a member of the set of fruit?

37:16 GREEN TRACK: No you are wrong. The cat cannot be a member of the set of fruit because a cat is not a fruit. Remember, the only members of the set of fruit are fruits.

YELLOW TRACK: Yes that's right. The cat cannot be a member of the set of fruits because a cat is not a fruit. You were right. Every member of the set of fruit is a fruit.

RED TRACK: The answer is no. The cat cannot be a member of the set of fruit because the cat is not a fruit. Every member of the set of fruits is a fruit.

38:30 Now you are looking at a set of balloons. All of the balloons together in a group make up the set of balloons. Have you ever had a balloon? This boy has lots of balloons. Each balloon that he is holding is a member of the set. All of the balloons make up his set of balloons.

39:38 This time the things we are going to talk about are coins. Have you ever been sent to the store to do some shopping? If you did you probably had a set of coins in your pocket. The coins in this set are a fifty cent piece, a quarter or a twenty-five cent piece, two dimes, a nickel, and three pennies. Each coin is one member of the set. Every member of the set is a coin. All of the coins together in a group make up the set of coins. You can see the different members of this set.

40:39 Here is a nickel. The nickel is a coin. Therefore, the nickel is a member of the set of coins.

41:40 Here is a dime. The dime is also a coin. Therefore, the dime is also a member of the set of coins.

42:737 This is a set of horses.

43:60 Is this cat a member of the set of horses?

44:49 If your answer is yes, press the green button. If your answer is no, press the yellow button. If you do not know, press the red button.

45:59 Remember the question, is this cat a member of the set of horses?

46:237 <u>GREEN TRACK:</u> No you are wrong. The cat cannot be a member of the set of horses because the cat is not a horse. Remember, the only members of the set of horses are horses.

<u>YELLOW TRACK:</u> Yes that's right. The cat cannot be a member of the set of horses because the cat is not a horse. You were right, every member of the set of horses is a horse.

<u>RED TRACK:</u> The answer is no. The cat cannot be a member of the set of horses because the cat is not a horse. Every member of the set of horses is a horse.

47:43 Here is a set of pretty color pencils. If you had these pencils you could draw and draw and draw. Every member of this set is a different color. Every member of this set is a pencil.

48:44 Here is one member of the set of pencils. There are lots of other members in the set of pencils. Can you think of one? Remember, all of the members of this set are pencils. Would a pen be a member of the set of pencils? Think about it.

49:1 Good-bye for now. I'll see you tomorrow and we'll do some more exploring in the basket of sets.

APPENDIX II

Description of the Slides Used in Lesson One

Slide Number	Description
1	Mr. Mathematics with basket of numbers, basket of sets, and basket of shapes
2	set of dishes
3	set of golf clubs
4	empty set
5	baby
6	set of swings
7	one of swings
8	set of swings / one of swings
9	set of pets
10	dog
11	cat
12	bird
13	set of pets / dog
14	set of pets / cat
15	set of pets / bird
16	set of fruit
17	apple
18	orange
19	banana
20	grapes
21	set of fruit / apple
22	set of fruit / orange
23	set of fruit / banana
24	set of fruit / grapes
25	set of five boys with different shirts
26	boy with white shirt
27	boy with purple shirt
28	set of five boys / boy with white shirt
29	set of five boys / boy with purple shirt
30	set of six balloons
31	blue balloon
32	set of balloons / yellow balloon
33	set of seven books of table
34	red book
35	green book
36	set of books / red book
37	set of books / green book
38	set of coins
39	nickel
40	dime
41	set of eight coins / nickel
42	set of coins / dime
43	set of nine pencils
44	blue pencil
45	set of pencils / red pencil
46	set of ten balls
47	one ball
48	set of balls / one ball
49	light cue: green light - yes; yellow light - no; red light - I don't know

(cont.)

Slide Number	Description
50	one of swings / light cues
51	bab / light cues
52	Mr. Mathematics frowning (perturbed)
53	Mr. Mathematics smiling (overjoyed)
54	Mr. Mathematics (befuddled)
55	swing / set of pets
56	baby / set of books
57	cat / fruit
58	nickel / pencils
59	cat / light cues
60	cat / set of horses

APPENDIX III

Pre-Mathematics Content Test (EMR)

Pre-mathematics Content Test

Name

1. Is a boy a member of the set of books?

YES NO

2. Is a boy a member of the set of fruits?

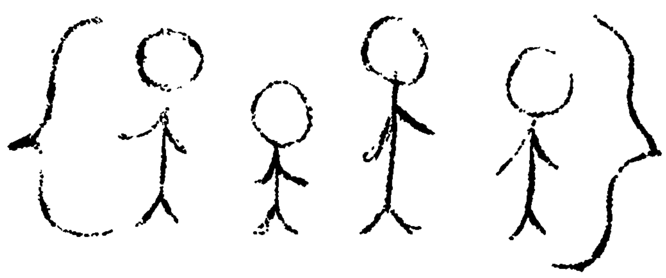
YES NO

3. Are flowers members of the set of horses?

YES NO

4. How many members are there in this set?

5. How many members in this set?

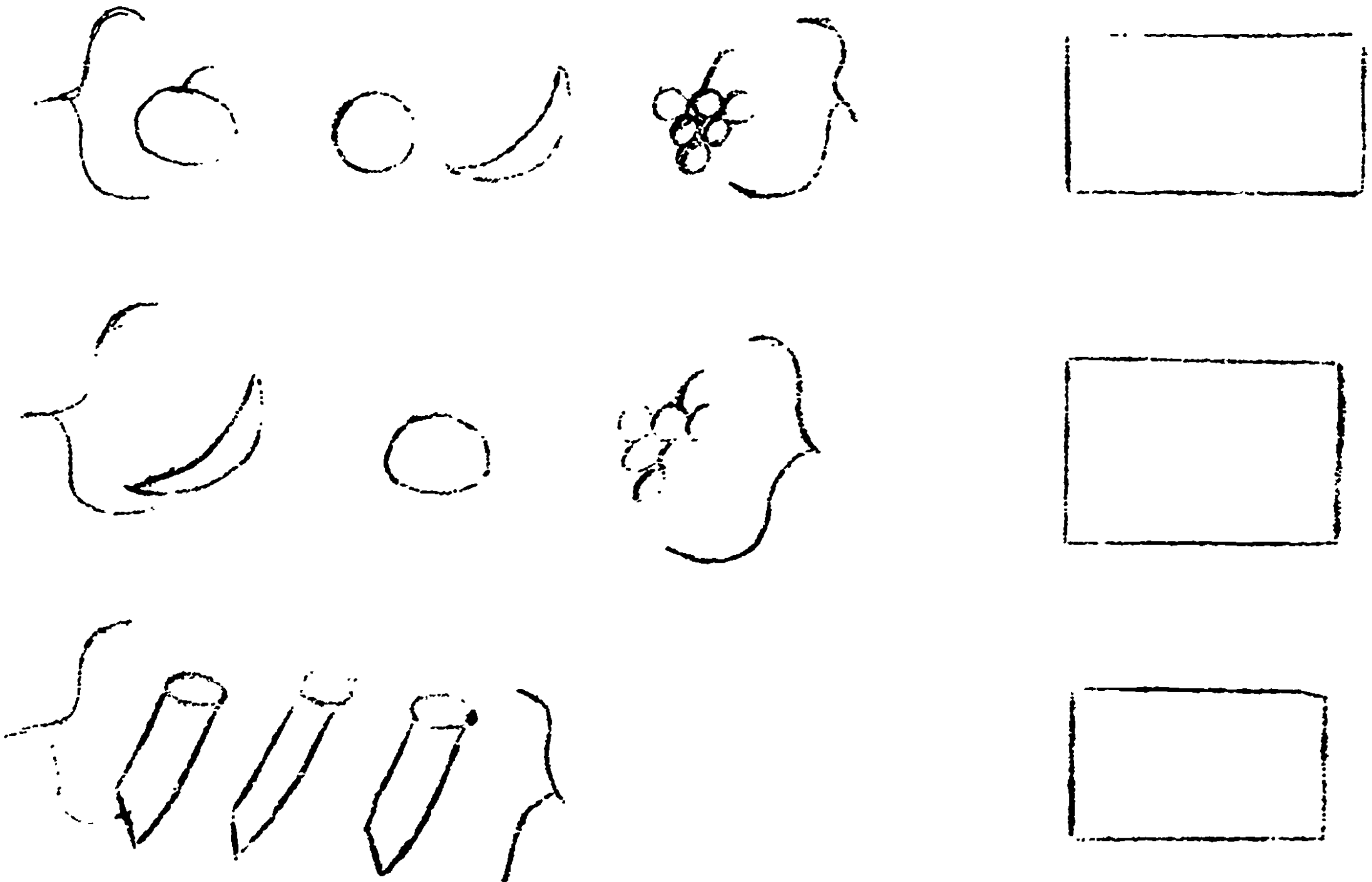


6. How many members in this set?

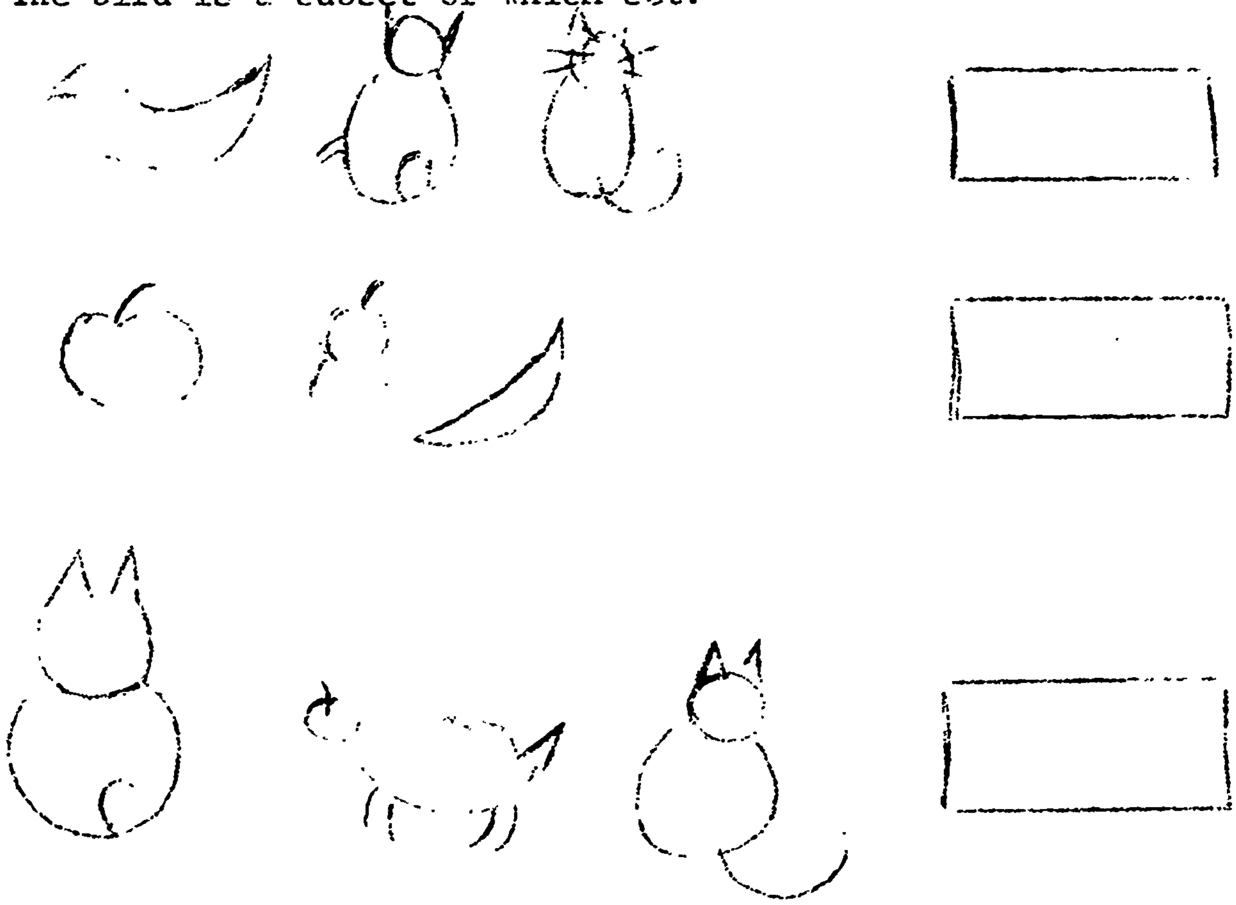
{ a b c }

____
Name

7. An apple is a subset of which set?



8. The bird is a subset of which set?



Name

9. Is a cup a member of the set of dishes?

Yes No

10. Is this {a, b, c, d,} a subject of this {a b}

Yes No

11. Is Wisconsin a subset of the United States?

Yes No

12. Is the United States a subset of Wisconsin?

Yes No

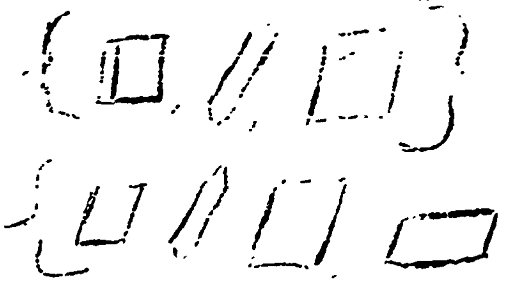
13. Are these sets equivalent?

Yes

No

Name

14. Are these sets equivalent?



Yes No

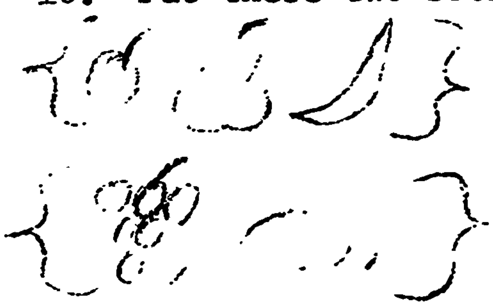
15. Are these sets equivalent?

{1, 2, 3, 4, 5, 6}

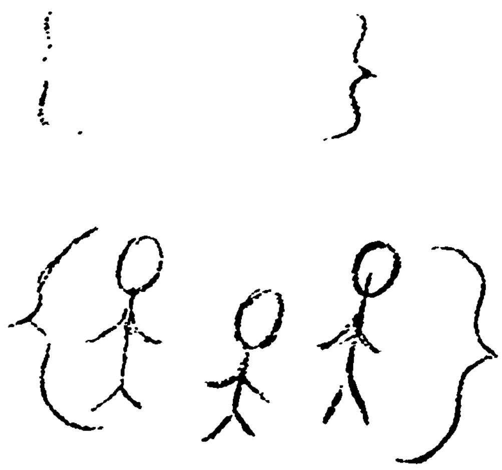
{a, b, c, d, e, f}

Yes No

16. Put these two sets together.



17. Union these two sets.



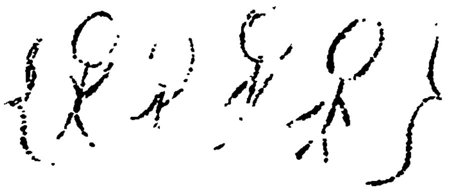
Name

18. Union these two sets.

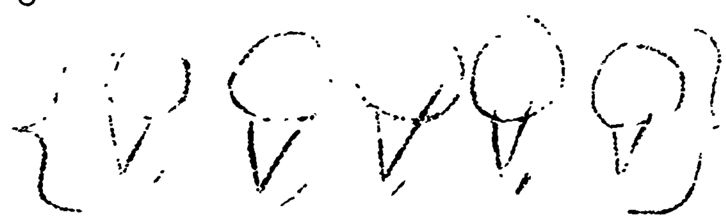
{1¢ 25¢ 1¢}

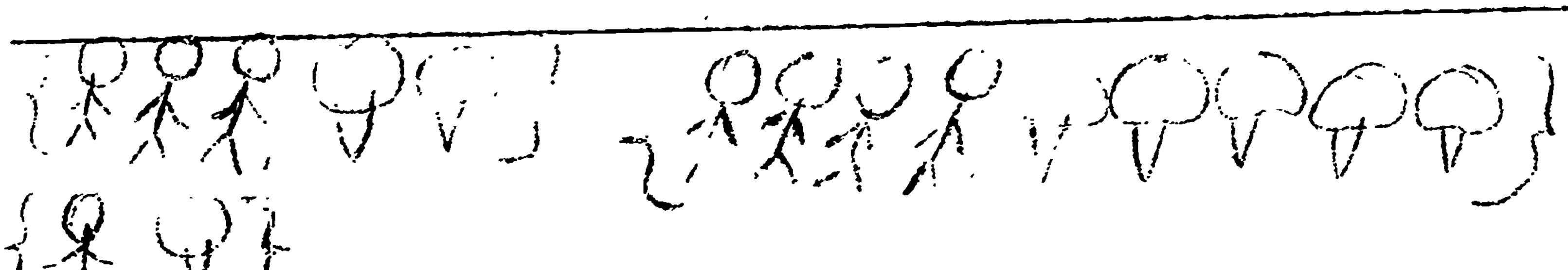
{5¢}

19. Cross out the new set we get from unioning this set



and this set.

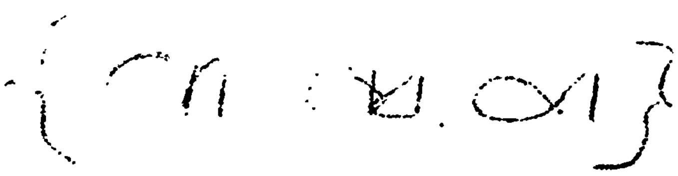


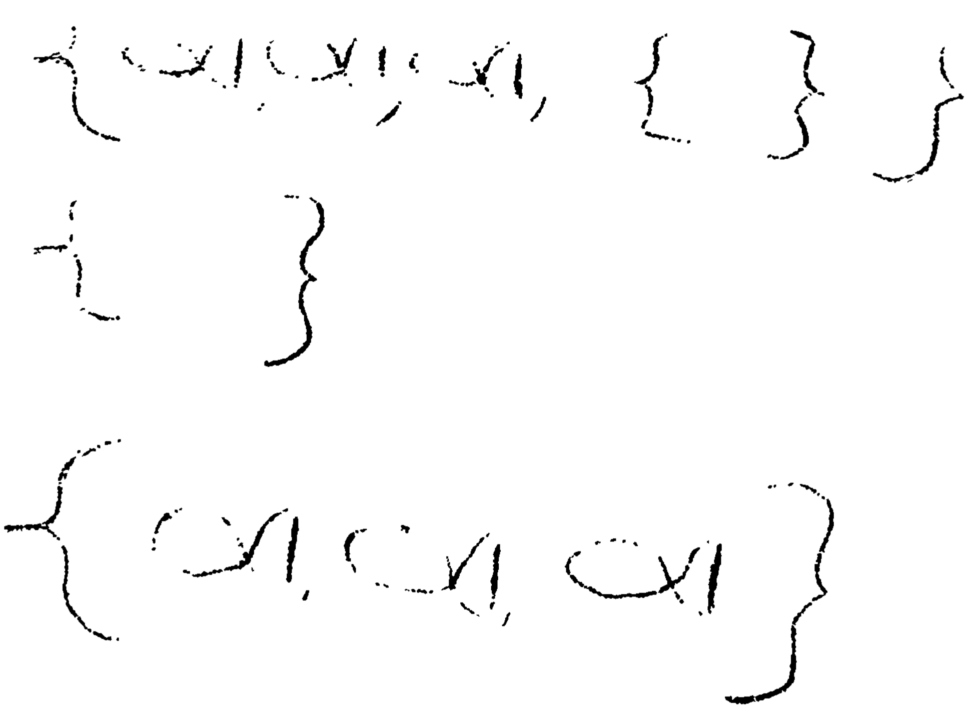


20. Cross out the new set we get from unioning this set

{ }

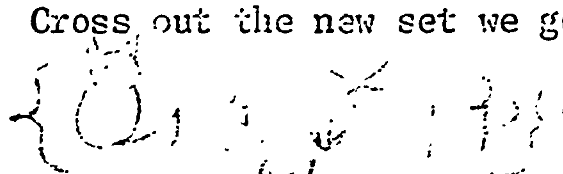
and this set.



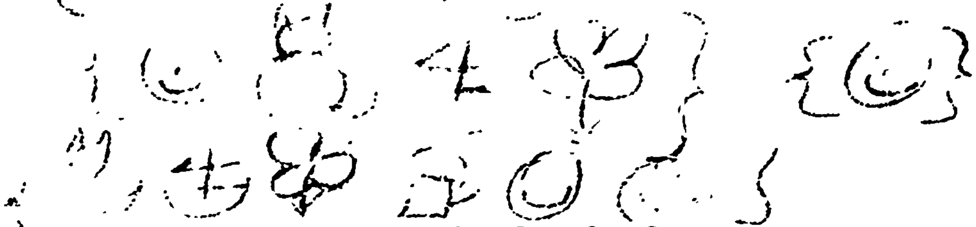


Name

21. Cross out the new set we get from unioning this set



and this set



22. What numeral is in the hundreds place?

123

23. What numeral is in the tens place?

[illegible]7

24. What numeral is in the units place?

163

25. Put the numeral 850 in the boxes below

hundreds tens units

Name

26. How many units make one ten?

27. How many tens make one hundred?

28. How many u its make one hundreu?

29. Does this x x x x x x x x x x equal 1 ten?

Yes No

30. x x x x x x x x x x x x x x = tens units

31. Is 5 equal to 5? Yes No

32. Is six pencils equal to seven pencils? Yes No

33. Can we say 9 does not equal 10?

Yes No

APPENDIX IV

Post-Mathematics Content Test (EMR)

POST-MATHEMATICS CONTENT TEST (EMR)

Practice Items:

1. Which of the following pets is a dog?
   a. (11) cat
   b. (12) bird
   c. (10) dog
   d. (304) fish

2. Which of the following is a set of horses?
   a. (738) set of three chickens
   b. (93) set of nine fish
   c. (737) set of four horses
   d. (81) set of three birds

---

1. A set is
   a. the number of members in a group.
   b. a group of things which go together.
   c. a dog, a cat, and a bird.
   d. any group of objects.

2. Which is not a set?
   a. (9) set of pets
   b. (4) empty set
   c. (120) group: a cat, a spoon, a sock
   d. (2) set of dishes

3. Which member would not be in the set of fruit?
   a. (17) apple
   b. (18) orange
   c. (20) grapes
   d. (44) pencil

4. Which set is the new set when you union these two sets: (370) empty set/ set of a firetruck and a policecar
   a. (4) the empty set
   b. (310) set of firetruck and policecar
   c. (311) set of fireman and policeman
   d. (370) set of firetruck and policecar/ the empty set

5. To union two sets means
   a. to put the members of the two sets together to make a new set.
   b. to make a new set by adding some but not all of the members of another set.
   c. to choose some of the members of a set and put them into a new set.
   d. to count the members in the sets.

6. Union these two sets: (373) set of faces/ set of hats Which is the new set?
   a. (371) set of faces
   b. (372) set of hats
   c. (374) set of faces and hats
   d. (151) set of blouses

7: Which object would not be in the new set when the set of fruit and the set of pie are unioned? (308) set of fruit/set of pie
   a. (79) pie
   b. (17) apple
   c. (19) banana
   d. (251) cookies

8. Which two sets were unioned to get this set: (306) set of dog, cat, bird, and fish
   a. (209) three black cats/black and white cats
   b. (13) pets/dog and cat
   c. (302) set of pets/fish
   d. (703) train/set of pets

9. Which two sets were not unioned to get this set: (364) set of a doll, a truck, and a ball
   a. (387) set of a doll, a truck, and a ball/empty set
   b. (388) set of a doll/set of a truck and a ball
   c. (389) set of a doll and a truck/set of a ball
   d. (369) set of a doll, a truck, and a ball/set of a boy and a girl

10. How many members in this set of books? (33) set of four books
   a. 8
   b. 4
   c. 7
   d. 6

11. The cardinal number of any set is
   a. the number of members in the set.
   b. the number which tells the order of the members in the set.
   c. the number which names the set.
   d. the number inside the set.

12. To find the number of members in a set, I would
   a. add the members.
   b. subtract the members.
   c. union the members.
   d. count the members.

13. Which of the following sets does not have eight members?
   a. (121) set of eight children
   b. (92) set of eight cars
   c. (33) set of seven books
   d. (38) set of eight coins

14. Which set does not have the same number of members as this set: (87) set of six hands
   a. (68) set of six footballs
   b. (72) set of seven airplanes
   c. (125) set of six balloons
   d. (270) set of six dowels

15. To union these two sets, I would put the (382) set of square and circle/set of a triangle
    a. circle into a new set by itself.
    b. square and triangle into one set.
    c. square, the triangle, and the circle together to make a new set.
    d. square into a new set by itself.

16. Which set is not a subset of this set: (9) set of pets
    a. (10) set of a dog
    b. (11) set of a cat
    c. (304) set of a fish
    d. (12) set of a bird

17. Which is not true about a subset? A subset is
    a. a set within a set.
    b. part of another set.
    c. a set with some but not all of the members of the set.
    d. a set with all of the members of the set.

18. A subset is
    a. a set which is part of another set.
    b. the number which tells how many members the set has.
    c. an empty set.
    d. a set which has the same number of members as another set.

19. Why is this group of objects not a set: (102) group: ball, dish, doll, cat
    a. A set can only have three members.
    b. The members of this group do not go together.
    c. The members of this group are different colors.
    d. A set does not have to have any members.

20. Equivalent sets are two sets
    a. with the same number of members.
    b. with the same members.
    c. with some but not all of the same members.
    d. which are the same length.

21. What member would not be in the set of pets?
    a. (9) dog
    b. (11) cat
    c. (12) bird
    d. (47) ball

22. This set is a set of (364) set with a doll, a truck, and a ball
    a. pets.
    b. balls.
    c. toys.
    d. cars.

23. Which pair of sets is equivalent?
    a. (373) set of faces/set of hats
    b. (261) set of helmets/set of heads
    c. (307) set of girls/set of boys
    d. (703) set of train cars/set of pets

4. A 2 in the tens place means we have how many things in our set?
   a. two, "2"
   b. twenty, "20"
   c. two hundred, "200"
   d. twelve, "12"

5. What are three places you can put these digets to write a numberal?
(503) set of digets 0 through 9
   a. units, sets, digets
   b. square, circle, tens
   c. units, tens, hundreds
   d. equivalent, nonequivalent, equal

6. Which numeral tells the number of members represented in this set of blocks?
(620) blocks which represent 154

		hundreds	tens	units
a.	(621)	1	6	3
b.	(617)	1	7	0
c.	(642)	0	5	4
d.	(620)	1	5	4

7. Which set of blocks represents this numeral? (629) 5 hundreds 0 tens 0 units
   a. (638) blocks which represent 900
   b. (630) blocks which represent 500
   c. (627) blocks which represent 400
   d. (631) blocks which represent 600

8. Which set does <u>not</u> represent this numeral? (482) 0 hundreds 1 tens 0 units
   a. (2) set of eight dishes
   b. (494) set of ten dots
   c. (498) set of ten sticks
   d. (46) set of ten balls

9. Which pair of numbers are <u>not</u> equal to each other?
   a. 4 and 4
   b. 5 and 5
   c. 7 and 8
   d. 6 and 6

0. What does this sign, = , mean? (2601) an equals sign
   a. is not equal to
   b. is equal to
   c. is a subset of
   d. is equivalent to